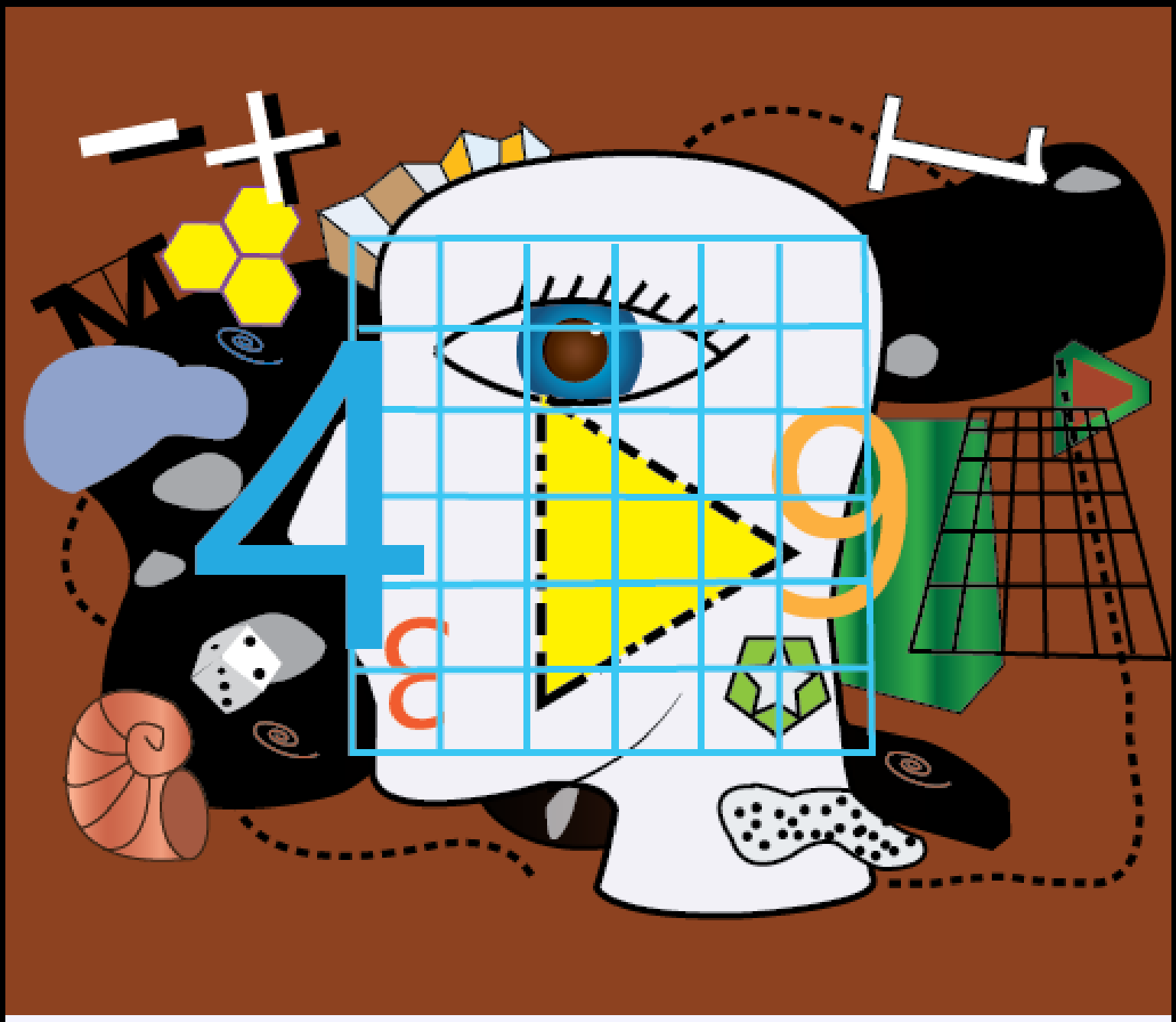


π-RATE

FOUNDERS' ISSUE 2021

CONTENTS

$$e^{i\pi} + 1 = 0$$

EDITORS' NOTE

Dear Readers,

Math, to most people, probably means a bunch of numbers, formulas or equations and even though that might seem the case at most times, its relevance in our life is immeasurable (though most things because of it aren't) once we start to take note of it-on our phones, social media algorithms, finances, space- even in the construction of our homes!

It is no wonder, with its enormous applications, that it has almost become second-nature to humankind: it has been around since forever and its formal study began as early as the 6th century BC. All this just makes us wonder- what exactly is left to do and do all these theorems and proofs even amount to anything substantial?

In the Founders' Day issue of the Pi-Rate, we have tried to include everything that we catch ourselves thinking of in the middle of a class, or while practicing before a test- what is the true nature of mathematics? Is it an invention or a discovery? Is it complete? Is it the absolute truth?

Through a range of articles ranging from the various paradoxes to Gödel's Incompleteness Theorem we have tried to answer all that, at least as far as math allows us to, and more. Additionally, we have tried to tackle misconceptions and explain why exactly you should be listening to music while studying math! This issue dives into what exactly makes math so difficult at times as well as its utilization in computers and art.

As first-timers, compiling and editing an entire magazine seemed like a daunting task but we were both thrilled at the prospect of being able to go beyond the textbooks and explore beyond the boundaries of the known, and we hope that we were able to make this issue just that- a thrilling experience!

Going forward, we encourage you to ask more questions, notice the smaller things and most importantly use your imagination!

Happy Reading!

Yours infinitely,

Shubhika and Tanvi

Editors-in-Chief

How is it possible that mathematics, a product of human thought that is independent of experience, fits so excellently the objects of reality?

–Albert Einstein

Hi [username], do you think your password was safe?

You must have come across sites that force you to use uppercase, lowercase, symbols and numbers in your passwords. While it can be frustrating, the truth is that by forcing passwords to be complex (by mandating certain character sets to appear) your password becomes lot harder to crack. Let's find out how strong your password is and how long would it take to hack it.

Theoretically, the number of possible passwords in a 4-digit numeric code would be 10*10*10*10=10,000 passwords, so the chances for a hacker to guess your password would be 1 out of 10000 but in real life people tend to select passwords they can remember rather than an arbitrary string of characters. We usually follow predictable patterns, even if it feels to us like we came up with random numbers. To make your password hard to crack, you must increase its sample space (number of possible passwords). Let's say your password is 'zuymsk'.

Here, the maximum number of possible passwords the hacker must attempt will be, 26(total number of alphabets) ^6(number of characters in the password) =308915776 but if we add 1 more digit to it, the number of attempts required will increase up to 26^7=8031810176. Now, if we exchange one alphabet by an uppercase letter, the sample space becomes (26+26) ^6 which is a much larger digit. Hence a longer password with a combination of different character sets is a lot stronger. An attack under these circumstances is called brute force attack. Here, the attacker is trying to guess your password from all the possible passwords in the sample space.

Now, we may think that 'password123' is better than 'whfos' as it is longer and has more character sets but in reality, it is a terrible password. In fact, a simple Dictionary attack can be used to hack such passwords. In this case, hackers use computational codes to create variations on top of words that people are most likely to use such as words they know, or passwords used before by other people.

To calculate the estimate time, it would take to crack your password, use the following formula:

A password with all lowercase characters like 'qwerty' would take almost 3 mins to crack, whereas a mix of lowercase and uppercase such as 'QwErTy' might take as long as 10 hours to crack just because its sample space increases.

In order for a password to be considered secure, it needs to be truly random and unique. For a unique password, we should never reuse it. Now the question arises, how do we remember 100 different truly random digits? We can use online password managers. These keep your passwords safe and can be only hacked by actually using your device. We can also use sites like HIBP which tell if your data has been compromised or the number of times a password has been used already. Therefore, if we're careful while creating our passwords, we can ensure our safety online by making sure that it remains uncrackable for thousands of years!

-Aaruni Garg
AII-B

Music & Math

"Music can change the world because it can change people."

-Bono

What if I told you that listening to music can actually help you improve your math skills? This is a direct allusion to the Right Brain-Left Brain theory, which argues that people are either left-brained or right-brained, with one side of the brain being dominant. You are said to be left-brained if you are mostly analytical and orderly in your thoughts and you're regarded as right-brained if you're more creative or artistic, that is, innovative. What's interesting about this theory, however, is that anybody can turn over a new leaf by putting both sides of their brain to work by playing music and solving problems at the same time! No wonder that even Einstein used to sit and listen to classical music when he was stuck with a mathematical problem!

More recent research has revealed that the Mozart Effect, which refers to the hypothesis that listening to Mozart's music can significantly raise scores on one element of an IQ test, has little to do with Mozart and more to do with listening to music that activates a specific part of the brain with the left hemisphere being initialized to process rhythm and pitch discrimination, whereas the right hemisphere for processing timbre and melody.

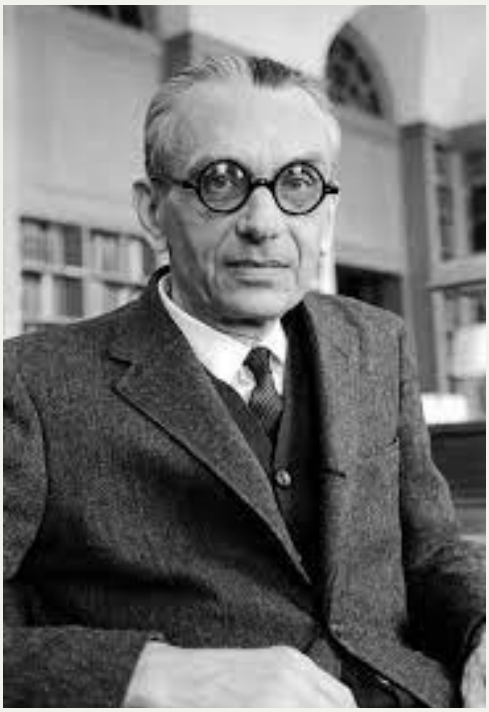
Dr. Robert Melillo has recently worked with a composer to create a series of music that focuses on strengthening each hemisphere- which you can utilize to study for tests!

In fact, one study published in 2012 found that listening to music while taking a math test can increase a student's cognition and ability to master arithmetic abilities by 40%. It was also discovered that youngsters who play instruments are better at solving hard mathematics problems than their counterparts who do not.

So, the next time you're about to give up on a problem or a sum- remember that music might help!

-Shambhavi Chandra
AII-A

GÖDEL'S INCOMPLETENESS THEOREM



Kurt Gödel

Consider the following words: "This statement is false."

This self-explanatory statement if assumed true, would mark itself as false. However, if it is false in this case, would lapse under the category of truth. Hence, by referring to itself, it creates an unsolvable paradox, putting up a heavy question upon its own validity. Though this theory seems silly, it did lead Austrian logician Kurt Gödel to a discovery that would change the heart of mathematics forever.

Every system built on mathematics, from the most complex proof to basic arithmetic, was constructed from axioms. At that time, the Mathematicians of the era sought a solid foundation comprising of a set of basic axioms, that were both consistent and complete- serving as the building blocks of all truths and laws. In other words, if a statement about numbers is true, mathematicians should be able to confirm it with an axiomatic proof.
When prominent mathematicians were focused on proving that their subject had no contradictions, Gödel was not so sure.
While it's relatively easy to create a self-referential paradox with words, numbers don't typically talk about themselves. Therefore, to experiment his hypothesis, Gödel first translated all mathematical statements into code numbers (known as Gödel Numbers), provided that each statement when written in such numbers reflected the encoded laws behind itself. Furthermore, Gödel gave every sequence of formulas a unique Gödel number too. In this way, he allowed entire equations of mathematics to talk about itself, and obtained a statement "The formula with Gödel number sub (y, y, 17) cannot be proved."

Now If the statement is false, the statement does have a proof. But if a mathematical statement has a proof, then it must be true.
Thus, the contradiction means that Gödel's statement can't be false, and therefore it must be true without any proof. Henceforth, we now receive a paradox right at the basic belief of mathematics, that is, a true equation that asserts it cannot be proved. This revelation rocked the foundations of the field- crushing those who dreamed that every mathematical claim would one day be proven or disproven.

Therefore, by such use of simple words in a tricky manner, the incompleteness theorem derived this logical subject from ever having a unification of what's provable and what's true.

-Bhavya Sangal
AI B

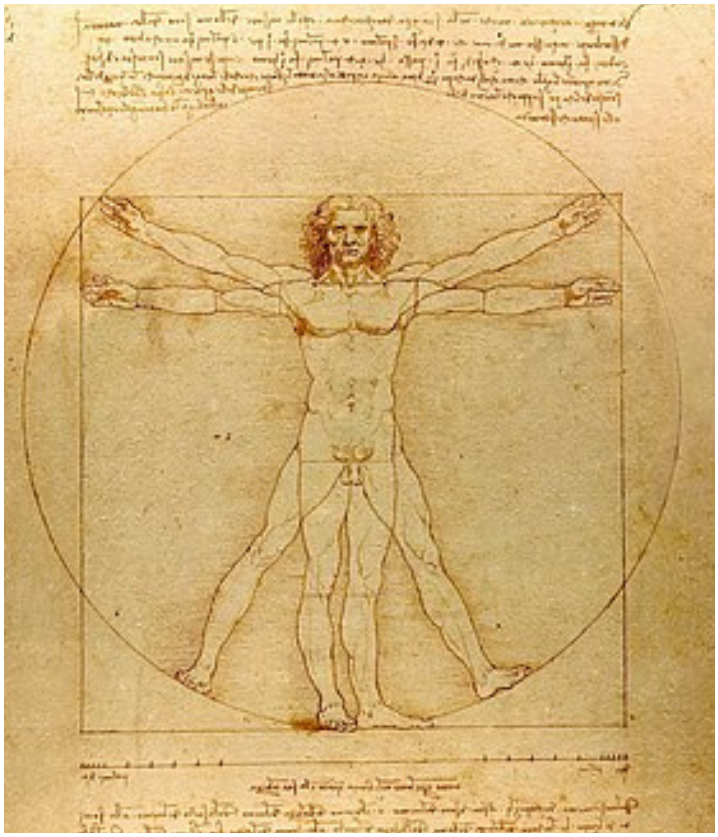
DID LEONARDO USE MATH IN PAINTING THE VITRUVIAN MAN?

“Painting is a science and all sciences are based on mathematics.”
~ Leonardo da Vinci

Leonardo Da Vinci is one of the most intriguing personalities in the art world. He was not only an artist- for which he is best known - but also an architect, scientist and mathematician. His works are a proof that mathematics, science and art work together to form beautiful brilliant masterpieces.

In his most famous sketch of the Vitruvian man, named after the architect Marco Vitruvius, Leonardo attempted to solve an age-old problem of “squaring a circle”.

Marco Vitruvius believed that if the naval was viewed to be the center of a human being, a compass could perfectly circumscribe a person with his arms outstretched. He also believed that for most people arm span and height are in perfect correspondence, thus placing the body inside a perfect square.



Using Vitruvius’s observations, Leonardo attempted to solve the problem of squaring a circle. We all know that area of a circle is and the area of a square is . Leonardo was attempting to use the area of the circle to create a square with the same area. The problem is based on Euclid’s elements of constructing a circle and square with compass and straightedge.

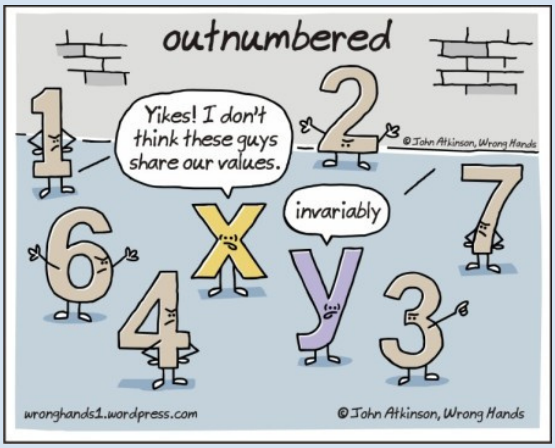
Leonardo’s solution with Vitruvian Man gives us a unique vision into the human body — one that’s obsessively symmetrical, deceptively simple, but extraordinarily beautiful. To create a circle and square with equal areas, he used the naval as the center of the circle and placed the Vitruvian man inside a square as claimed by Vitruvius that arm span equals height. He considered the area of the human body equal to the area of both the square and circle since it perfect fit within both shapes.
Though because of the irrational nature of pi, Leonardo wasn’t entirely successful in solving the problem, however he did come very close in solving it!

-Tanvi Agarwal
Pre-Sc B

Math Mirth

SUDOKO CHALLENGE!

	2	6		3				8
9			6			1		
				1	9		4	
		7	3		2			
		4		7		8		
			8		6	7		
	5		7	2				
		9			5			4
4				6		2	1	



Jokes for you to laugh!

- Why did the girl wear glasses during math class?
  It improved di-vision.
- Why did Pi get its driver's license revoked?
  Because it didn't know when to stop.
- There's a fine line between a numerator and a denominator… But only a fraction would understand.

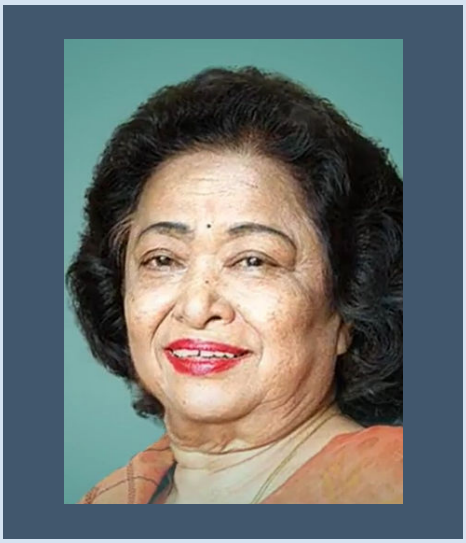
ANSWER:

1	2	6	4	3	7	5	9	8
9	4	3	6	5	8	1	2	7
7	8	5	2	1	9	3	4	6
8	6	7	3	9	2	4	5	1
3	9	4	5	7	1	8	6	2
5	1	2	8	4	6	7	3	9
6	5	1	7	2	4	9	8	3
2	3	9	1	8	5	6	7	4
4	7	8	9	6	3	2	1	5

MATH-A-GENIUS

SHAKUNTALA DEVI

"Education is not just about going to school and getting a degree. It's about widening your knowledge and absorbing the truth about life."



These lines were once said by the famous and legendary mathematician Shakuntala Devi who was a writer and mental calculator, popularly known as the "Human Computer." In 1950, she embarked on a tour and completely astounded academics and the public all over Europe by her amazing computational abilities.

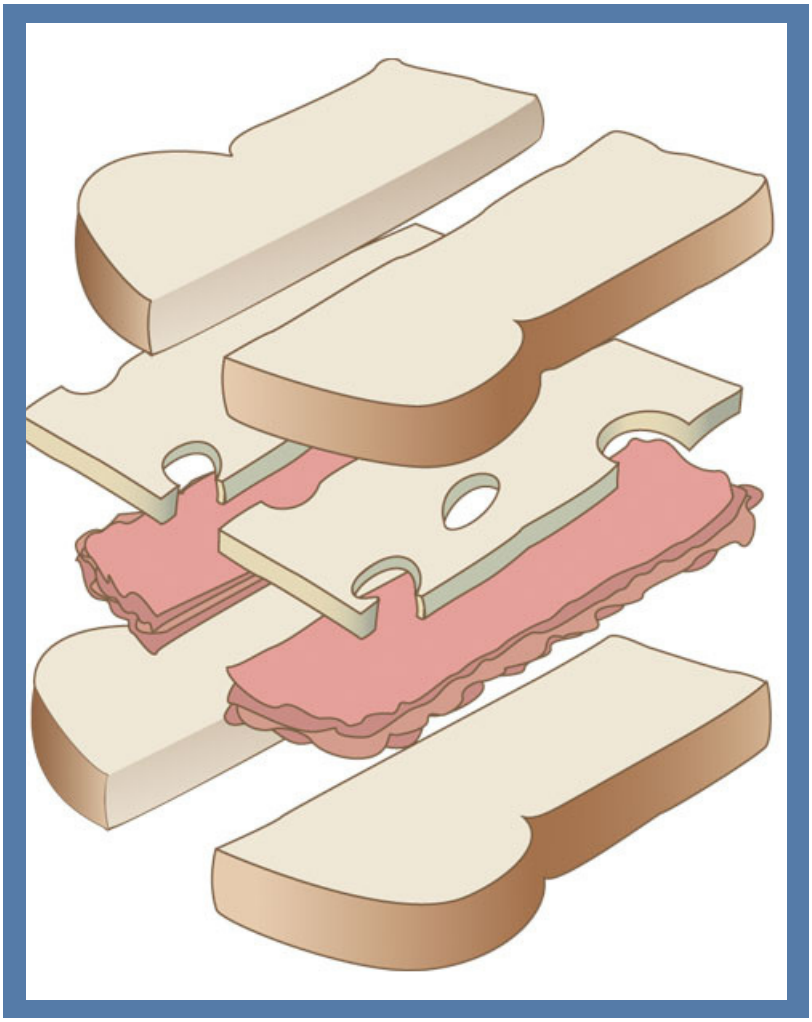
It was around this time during a BBC appearance that she was conferred the title of "Human Computer." She travelled to several countries around the world demonstrating her arithmetic talents. In Southern Methodist College in Dallas, she calculated the 23rd root of a 201-digit long number in less than a minute whereas a UNIVAC 1101 computer took 12 seconds longer than her to do the same.

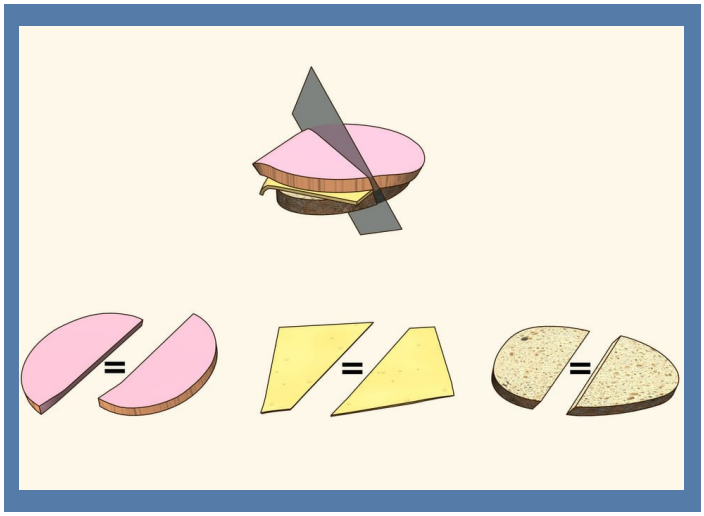
On 18 June 1980, she demonstrated the multiplication of two 13-digit numbers – 7,686,369,774,870 × 2,465,099,745,779. These numbers were picked at random by the Department of Computing at Imperial College London. She correctly answered 18,947,668,177,995,426,462,773,730 in 28 seconds. This event was recorded in the 1982 Guinness Book of World Records. According to Professor Arthur Jensen, in a research study published in the journal Intelligence in 1990: "Devi solved most of the problems faster than I was able to copy them in my notebook." Jensen set her two problems, the cube root of 61,629,875, and the seventh root of 170,859,375. Shakuntala Devi gave the correct answers — 395 and 15 — even before Jensen's wife could start the stopwatch.

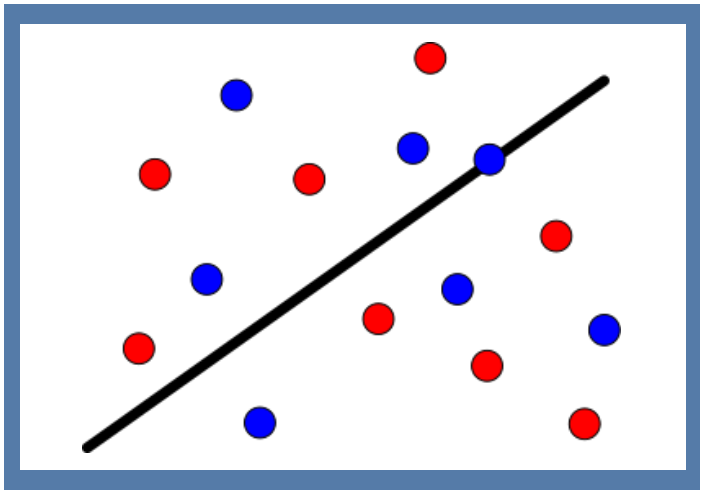
Shakuntala Devi won the 'Distinguished Woman of the Year Award' in 1969, from the University of Philippines along with a gold medal. In 1988, she was honoured with the 'Ramanujan Mathematical Genius Award' in Washington D.C., conferred to her by the-then Indian Ambassador to US. Her name was listed in the '1995 Guinness Book of World Records' edition. She is the first Indian woman who studied homosexuality and wrote a book on the same subject titled 'The World of Homosexuals'. She also wrote several other books which completely portray her mathematical prowess such as 'Astrology for You', 'Books of Numbers' and the infamous 'In the Wonderland Figuring: The Joy of Numbers'.

On 21 April,2013 the world suffered a loss of one of the greatest mathematician there has been and on 21 July 2020, a movie was released in her memory and the role was gracefully played by Vidya Balan Shakuntala Devi truly was and will always be remembered as a legend in the whole world.

-Pranya Dua
AIII C







The Ham Sandwich Theorem

There's a theorem that says it's always possible to make a cut through a ham and cheese sandwich so that the two halves have exactly equal amounts of bread, ham, and cheese. The ingredients may be any shape you wish, and even in different places – say, the bread inside the bread packet, the cheese within the fridge, and the ham on the counter. They might even be scattered across the galaxy!

The reasoning for this is that there is a version that in N-dimensional space, any N globs of positive volume can be simultaneously bisected by a single hyperplane. The Ham Sandwich Theorem can be proved using the Borsuk-Ulam theorem. What's that you may ask? It is a theory which says that at any given moment on the earth's surface, there exist 2 antipodal points (on exactly opposite sides of the earth) with identical temperature and barometric pressure!
This theorem holds true every single time. A slice made – or a plane drawn – can equally divide the bread, ham, and cheese. The theorem even holds in higher dimensions, for example, in five dimensions, five objects, whatever their shape and position, can always be bisected with a single slice.

-Tamana Baid
AIII C

Most Beautiful Equation

“Gentlemen, that is surely true, it is absolutely paradoxical, we can’t understand it, and we don’t know what it means, but we have proved it, and therefore we know it must be the truth.”

-Benjamin Peirce(19th CE)
Harvard mathematician

Euler's identity is an equality found in mathematics that has been compared to the Shakespearean sonnet and described as "the most beautiful equation", consisting of the five most important mathematical constants and can be written as :

$$e^{i\pi} + 1 = 0$$

Euler's identity arises naturally from the interaction of complex numbers which are numbers consisting of two parts: real numbers and imaginary numbers, for instance, 4 + 3i. Complex numbers appear in many applications such as wave mechanics and circuit designs. Furthermore, complex and hypercomplex numbers have properties that make them especially useful for studying computer graphics, robotics, navigation, flight dynamics, and orbital mechanics: multiplying them together makes them spin. This property helps us obtain a clear understanding of the reason behind Euler's identity.

One of the most necessary elements of this equation is the constant ‘e’, also referred to as the Euler’s number, which has a value approximately equal to 2.71828. This transcendental element is used in the calculation of a number of important concepts that include compound interest and for the computation of exponential growth and decay across a wide variety of applications.

This formula really comes into play when we have to solve differential equations with constant coefficients. So the goal is to find the right numbers a, b so that the above functions that we just differentiated solve the given equation. For example, electrical circuits lead to differential equations relating current, charge, and voltage based on circuit elements. The circuit elements are described by several parameters such as inductance, resistance and capacitance. This becomes the coefficient in the differential equation.

Euler’s formula has not only enabled us to write the exponential functions in terms of sine and cosine, the linear combinations of exponential functions with imaginary exponents but also to find a formula for the sum of exponential functions, cos z and sin z. It has made the conversion of complex numbers from one form to another a lot more convenient and has been greatly praised by mathematicians all over the world.

-Avani Jindal
Pre-Sc B

Philosomatics

Mathematics- 1,2,3... a2+b2+2ab, H2=P2+B2
While the pictures that popped up in your head when I said the word Mathematics were concrete- probably something similar to what I've mentioned above, the establishment of the subject itself was very abstract. In fact, some would say it still remains abstract.
Perhaps the subject that is most connected to this concept of the "ABSTRACT", however, is philosophy- even though most of us seem to think of it as this supreme subject that only great scholars pursue.

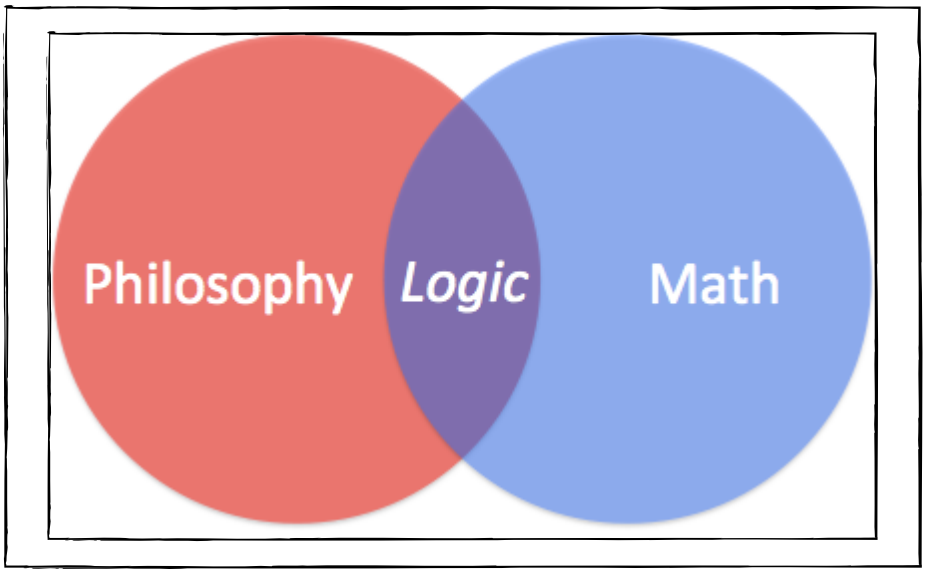
To address your wonder regarding this unusual combination of subjects I've mentioned- surprisingly, mathematics and philosophy have a lot in common. To begin with, both subjects were established on the basis of logic. Initially, they were considered two different methods altogether, since then, however, many like Galileo have been trying to bridge the gap.

Logic and Mathematics go hand in hand. 1+1=2 is a mathematical conclusion we all agree with based on the rules of addition and subtraction- hence it is logical. But so are philosophical ideas. Though they are ideas, their core lies in logic.

Philosophy and mathematics are also integral to each other. Mathematical principles help establish the philosophy of various disciplines from sciences to humanities. Philosophy too is essential for understanding the methods used in mathematics and how those methods affect other disciplines and our daily lives.

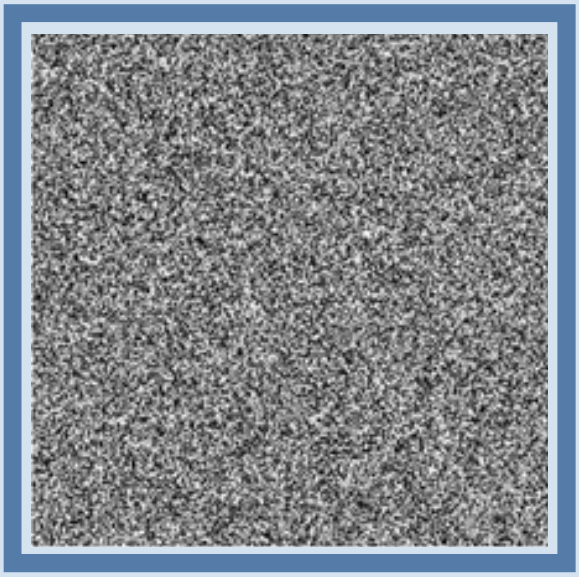
To put it simply, you can think of them as the chicken(philosophy) and the egg(mathematics). Which gave birth to the other or what came first? We may never know. However, what we do know is that mathematics and philosophy are interconnected, with logic being their strongest link.

-Krisha Khanna
Sc B

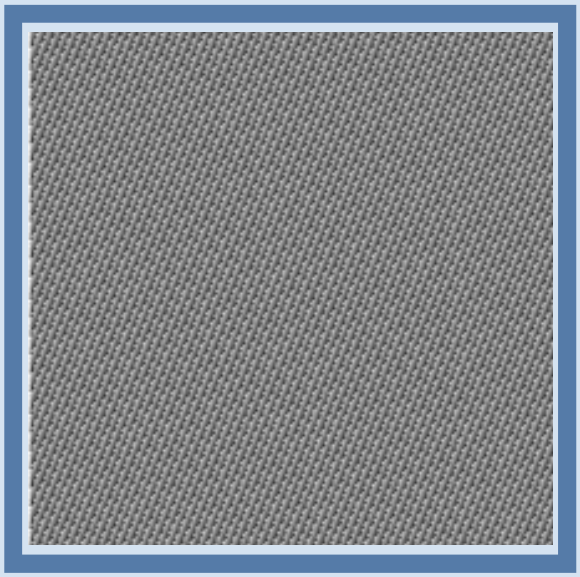


ARE RANDOM NUMBERS REALLY RANDOM?

Randomness is a part of nature. Predicting the direction in which leaves will fall, or the patterns that clouds will make in the sky is nearly impossible. However, some things are not as random as they seem. We've all used random number generators on the internet. Though it may look like it, these generated numbers are not really random, and can actually be predicted. Random numbers supplied by a computer are generated by mathematical formulas, and so by definition cannot be truly random.



True Random Numbers



Pseudo Random Numbers

The random number classes or libraries that computers use in their code are Pseudo-Random Numbers - they behave like they are random but in reality, they are not. Essentially, PRNGs (Pseudo-Random Number Generators) are algorithms that use mathematical formula or simply precalculated tables to produce sequences of numbers that appear random. PRNGs use an initial value known as a seed value to determine the end results. Therefore, if you know the seed value and the algorithm used (a sequence of mathematical operations for example), it is easy to reproduce these seemingly random results.

True random numbers can only be obtained by services that generate randomness via physical phenomenon. The physical phenomenon can be quite simple, like the little variations in somebody's mouse movements on a keypad, or more complex like radioactive decay and its spread. The hardware part of the service consists of a device that converts energy from one form to another (for example, radiation to an electrical signal), which then provides an output of a digital number. Random numbers generated based on such randomness are said to be "true" random numbers, as they cannot be predicted.

Even though scientists are working towards predicting patterns for the more serious problems like nuclear decay, in the end the true values will always remain 'random.'

-Anushka Prakash
Pre-Sc B

Why do math tests make me anxious?

If staring at a complicated math equation makes you want to turn the page and sitting in a math test makes you freeze up, chances are that you suffer from math anxiety. Mathematical anxiety refers to the fear or apprehension of mathematics or mathematics-related tests. This form of anxiety makes one's mind go blank when they have to handle situations with numbers and induces extreme stress.

How does one develop such panic only about math though? The core of this answer lies in the subject, from childhood, being associated with a sign of intelligence and meritocracy, and this ideology being enforced by parents and teachers alike. When a student slips in math even once, they are often faced with an embarrassing situation where their tiny error is almost equated to intellectual inability. This social dismissal and pain (due to high performance in any circumstance being the sole focus of capitalism) make the students believe that one can only naturally be good at math, and that it is purely genetic.

This leads to another myth that one cannot improve at math, and it all depends on one's genetic composition. However, math- like any other subject, can be improved upon with a growth mindset and the right environment. Sadly, with the fast pace and daunting atmosphere that math classes often take on, students do not end up exploring their potential.

During a test, students make use of their working memory. This working memory allows them to access relevant information required to answer a question, be it the formulas or the techniques.

When one is trapped by math anxiety, their anxiety shrinks the working memory's capacity, inhibiting the student from performing to the maximum of their abilities. derwent during their social dismissal.
When a teacher sees rushed calculations and basic arithmetic errors, it is not the student being lazy but a mechanism for them to avoid the same pain they underwent during their social dismissal.

However, it is important to remember that math anxiety is not inevitable. Battling this issue can begin with everyone unlearning the concept that excelling at math is the primary indicator of intelligence because everyone can be good at it provided that they give it the time it needs, and do not go in with preconceived notions.

-Anjali Agarwal
Sc A

ZENO: WHY?!

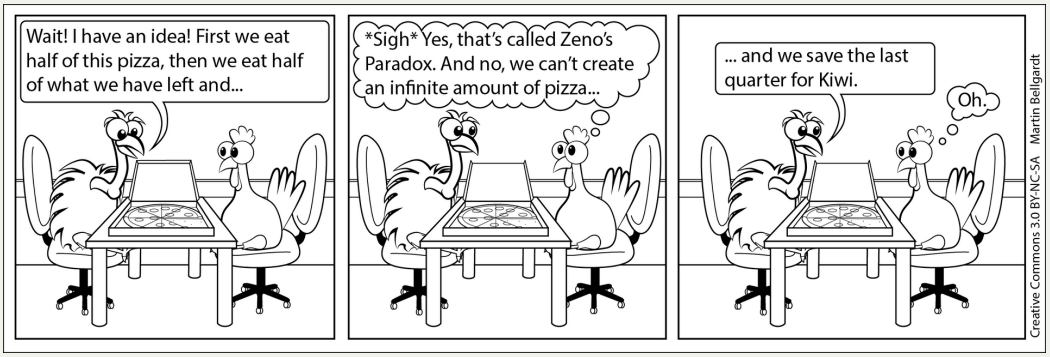
Calculus, considered one of the most tedious branches of mathematics, is the “calculation of instantaneous rates of change (differential calculus) and the summation of infinitely many small factors to determine some whole (integral calculus)”.
Even though you might dread the subject, the real-life applications of the field are enormous- right from launching satellites in space to predicting weather patterns and calculating the rate of spread of a disease.

It might have crossed your mind how these peculiar theorems came to be (whether due to curiosity or in despair)? The answer lies in Zeno’s paradox- which is considered, perhaps, one of the earliest examples of proof-by-contradiction stating that “In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead”. To elaborate, if Achilles and a tortoise are in a footrace(the idea sounds absurd but bear with me) with the tortoise getting a head-start of about 100 metres, for example, even with Achilles running way faster than the tortoise he should never be able to catch up as the tortoise covers an infinitely small distance as soon as Achilles reaches the spot that the turtle started travelling from initially as both are constantly moving.

Similarly, during the motion of an arrow in an instant of time the arrow can be captured to be still in air(suppose with a camera). However, since the total time is composed of these instants and there is no motion in considered instant- does the arrow remain in rest after all? Likewise, is the dichotomy paradox which states that it is impossible to cross a path as we would first be required to cross half of it, then another half(or a quarter of the whole) and similar other halves of the uncovered distance- leading us to take infinite steps practically. The second conflicting conclusion derived from the same was that since any distance can be divided in half, the motion must be an illusion as it never started or ended for a finite distance.

Although the answers to these might seem obvious practically- mathematical proofs took centuries to develop even with great scholars such as Archimedes, Aristotle and Bertrand Russel trying their hands at it- the fatal flaw in their devised theorems being that they assumed Zeno’s procedure was correct. Finally, it was Leibniz and Newton who “independently” developed the conclusive rules of calculus- which ultimately lead to one of the biggest mathematical controversies till date, but that is a story for another day.

-Shubhika Khanna
Pre-Sc C



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